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Time-evolution of cascade processes of muonic atoms in hydrogen-helium mixtures

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Abstract. The time-dependence of the population of muonic hydrogen states in hydrogen-helium mixtures is calculated for principal quantum number n. The number of muons transferred to helium nuclei is also determined. The dependence of the population of the ground state of muonic hydrogen q_{1s}^{He} on time and target density and the helium concentration is also considered. The results are in agreement with recent experimental data. The comparison of the calculated yield of K lines of X-ray in pure hydrogen and deuterium with experimental data indicates the essential role of the Coulomb deexcitation process. Possible Stark mixing is also analysed.

PACS. 34.60.+z Scattering in highly excited states (e.g. Rydberg states) – 34.70.+e Charge transfer – 36.10.-k Exotic atoms and molecules (containing mesons, muons, and other unusual particles)

1 Introduction

The processes induced by negative muon stopped in mixtures of hydrogen isotopes with admixtures of $Z > 1$ elements have been studied for some time already; however, some interesting questions about the processes remain open. Research of nuclear fusion in charge-asymmetric muonic molecules like $h\mu Z$ ($h \equiv p, d, t$ is a hydrogen isotope, Z is an isotope of helium, lithium, beryllium etc.) allows extending the energy region of investigation of strong interactions to very low energies (eV ÷ keV), which cannot be reached in accelerator experiments. The properties of strong interactions such as charge symmetry or isotopic invariance are all experimentally established mainly in the MeV region and, up to now, have only been extrapolated to the low-energy region [1].

The study of fusion reactions between light nuclei is important also for astrophysics. Specifically, this study is relevant to the nuclear reactions which took place in the process of primordial nucleosynthesis just after the Big Bang, and those occuring in stars, where light elements are produced. For example, in stars and in the Galaxy one finds a deficiency of light nuclei (except for ^{4}He) compared with predictions based on the theory of thermonuclear reactions and generally adapted models. To explain this phenomenon, modified star models are usually proposed, which assume that in the extrapolation of nuclear cross-sections from accelerator energies to the astrophysical energy region ($\sim$ keV) no resonances or other anomalies of the cross-sections occur. It cannot be excluded, however, that nuclear cross-sections have a resonance character, which could lead to intensive burning of light elements in stars [1].

Most theoretical and experimental studies of asymmetric muonic molecules have been devoted to the $h\mu$He systems. Recently, the dynamics of muonic atom cascade in hydrogen-helium mixtures was considered in [2]. This is important [3] for the investigation of the nuclear fusion reactions[1]:

$$d\mu^3\mathrm{He} \xrightarrow{\lambda_\mathrm{f}} \begin{cases} \alpha + p(14.7\ \mathrm{MeV}) + \mu \\ ^5\mathrm{Li} + \gamma(16.4\ \mathrm{MeV}) + \mu \end{cases} \tag{1}$$

and $d\mu^4$He

$$d\mu^4\mathrm{He} \xrightarrow{\lambda_\mathrm{f}} {}^6\mathrm{Li} + \gamma(1.48\ \mathrm{MeV}) + \mu. \tag{2}$$

However, the time-evolution of the cascade processes was not considered there. The experimental investigation of fast processes in muonic atoms became possible due to development of short-time techniques and the essential

[1] Taking into account the importance of reactions (1) and (2) we shall mainly consider the dynamics of muonic cascade in deuterium-helium mixtures.

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improvement of the time-resolution of detectors for registration of nuclear radiation. The experimental information about the characteristics of the cascade of muonic hydrogen could be obtained as a rule by the measurement of yields and time-distributions of the muonic X-ray of the hydrogen isotopes and $Z > 1$ admixtures in hydrogen targets[2]. Therefore, the comparison of experimental and theoretical yields and time-distributions of the muonic X-rays may allow one to test the description of the real muonic cascade. Consequently, the measurements of intensity of delayed K-lines could enable one to determine the population of the $2s$-state of muonic hydrogen as well as the scheme of the cascade used in the calculations.

The measurement of X-ray yields of muonic helium in hydrogen-helium mixtures enables one to obtain the muon transfer rates from hydrogen to helium (in comparison with experimental data obtained in pure hydrogen mixtures) and to check the scheme of the cascade. The comparison of the experimental and calculated relative intensities of K-lines, *i.e.* K_α/K, K_β/K and K_γ/K (where $K = K_\alpha + K_\beta + K_\gamma + K_\nu$) as well as the ratios of intensities K_α/K_β, K_β/K_γ etc. in pure hydrogen mixtures and those containing $Z > 1$ admixtures, can allow one to investigate the dynamics of muonic cascade. The determination of population of excited states of muonic hydrogen and muonic helium in hydrogen-helium mixtures by measurement of the corresponding X-rays may let one to verify the scheme of muonic cascade as well as muon transfer process.

The analysis of time-evolution of muonic cascade should allow one to choose the experimental conditions for which the cascade model can be verified with theory. Such time-evolution data can give very important additional information about exotic systems.

In the present paper the time-dependence of population of excited levels of muonic atoms during the cascade as well as the dependence of muon transfer rates to helium nuclei on target density ϕ and helium concentration $C_{\rm He}$ are presented.

2 Cascade model

The number of $d\mu$He molecules formed in DHe mixture is proportional to the number of muonic deuterium atoms, dμ, in the ground state, which is determined by the probability [2]

$$W = W_{\rm D} q_{1s}^{\rm He}, \tag{3}$$

where $W_{\rm D}$ is the probability of formation of $d\mu$ atom in the excited state (with principal quantum number $n \simeq \sqrt{m_\mu/m_{\rm e}} = 14$) and $q_{1s}^{\rm He}$ is the probability of deexcitation of the muonic deuterium to its ground state (ground state population)[3]. Following [2] we determine

$$W_{\rm D} = \left(1 + \frac{C_{\rm He}}{C_{\rm D}} A\right)^{-1}, \tag{4}$$

where $C_{\rm D}$ is deuterium concentration ($C_{\rm He} + C_{\rm D} = 1$) and A is the ratio of the muon capture rates for helium and deuterium: $A = 1.7 \pm 0.2$ [2] (the analogous ratio for protium and deuterium is equal 1.204 in $H_2 + D_2$ mixture [4]).

As shown in the previous investigations, the ground state of muonic atom is reached only by a fraction of muonic hydrogen atoms – formed in an excited state – owing to muon transfer to heavier hydrogen isotopes or to $Z > 1$ nuclei, *e.g.* to He. As the cascade evolution occurs in a very short time ($\leq 10^{-11}$ s at liquid hydrogen density, LHD, $N_0 = 4.25 \times 10^{22}$ cm^{-3}) on the scale of muonic atom processes, the experimental information about the numerous processes occuring during deexcitation of the muonic atom is poor. At the same time, the theoretical predictions, especially concerning the early stages of the muonic cascade are also not fully determined [5]. Therefore, the research of alternative methods of obtaining information about W and the corresponding $W_{\rm D}$ and $q_{1s}^{\rm He}$ is very important. The agreement between experimental and theoretical data obtained using analogous assumptions about the cascade scheme for pure hydrogen targets and hydrogen-helium mixtures could provide a basis to extend this scheme to a consideration of time-evolution processes.

The basic cascade processes are the following [2,6,7]: radiative and Auger deexcitation, Stark mixing, Coulomb deexcitation, elastic scattering responsible for the thermalization of the muonic hydrogen and muon transfer from excited muonic hydrogen to other nuclei. According to [2,6] we assume constant collision energy $\varepsilon = 0.04 \div 2$ eV of the muonic atom during the cascade. As the accurate calculation of Auger and Coulomb deexcitation rates on helium nuclei are still absent, as well as the deexcitation rates of $2s$ level owing to Stark mixing with $2p$ level in the field of helium nucleus, we follow the approximation used in [2]. The corresponding Auger rates are approximated by $2\lambda_{\rm A}$, where $\lambda_{\rm A}$ are calculated according to formula (8) of reference [8] with replacement of ionization energy of the hydrogen atom by helium ionization energy, $I_{\rm He} = 24.68$ eV (see Tab. 1 of [2]). This simplest approximation is reasonable for our consideration. Factor 2 corresponds to two electrons of the helium atom in comparison with one electron of the hydrogen atom (although an effective charge of helium nucleus acting on conversion electron differs from 1).

The reaction rates for Coulomb deexcitation, Stark mixing, transitions between $2s$ and $2p$, and induced $2s \to 2p \to 1s$ transitions are supposed to be the same as those for collisions of muonic deuterium with tritium and are taken from [6,9], respectively. The rates of muon transfer from muonic deuterium to helium nuclei for $n \leq 5$ have

[2] In fact, information about dynamics of cascade could be improved by development of Auger spectroscopy. The measurement of time distributions of electrons of muon decay in muonic hydrogen could give additional information about muonic atom cascade in mixtures of hydrogen isotopes with $Z > 1$ admixtures.

[3] The index "He" was added to underline that hydrogen-helium isotopic mixtures are considered.

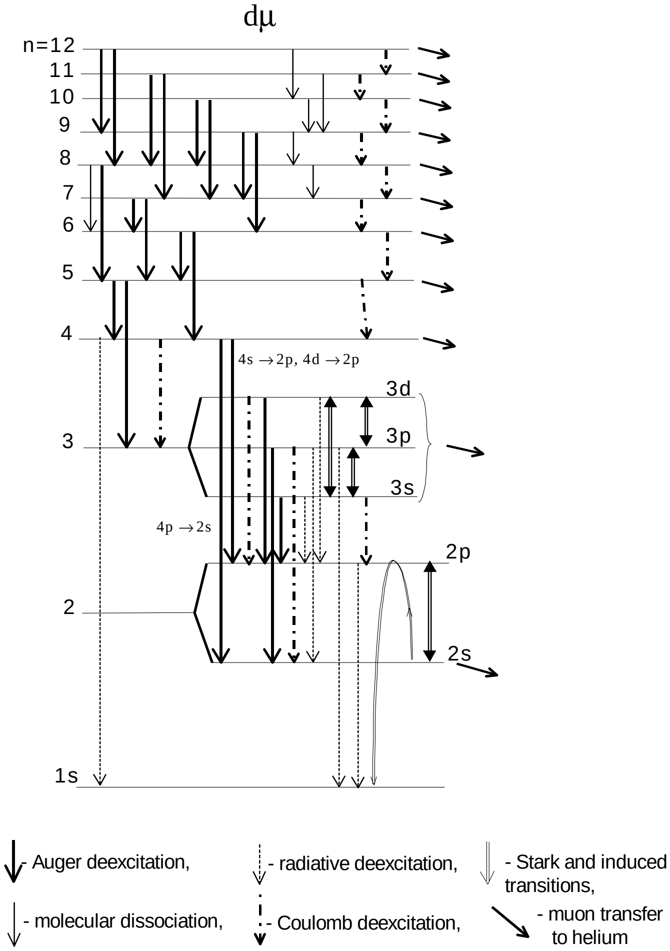

Fig. 1. Scheme of the muonic atom cascade for muonic deuterium.

been calculated in [10]. As there are no theoretical data for muon transfer to helium from $n > 5$, the corresponding values supposed to be equal to the transfer rates from $n = 5$ [2]. As follows from our calculations the observables ($q_{1s}^{\rm He}$ and relative intensities of K-lines) depend weakly on muon transfer from high levels because of fast deexcitation.

According to [5] we suppose that the initial n distribution is peaked around $n = 12$, so our calculations of the cascade parameters are based on the solution of a system of kinetic equations with the initial population of the state $q_{12}^{\rm He} = 1$ for the cascade scheme presented in Figure 1 with radiative and Auger transitions taken from Table 1 in [2] and Table 15 in [11] recalculated for the muon mass. We take into account the $3 \to 2$ Coulomb transition (not considered in [2,13]) and Stark transitions between the sublevels with orbital angular momentum l for $n = 3$ *i.e.* between $3d$, $3p$ and $3s$ [12][4]. We assume statistical population of l-sublevels for $n > 3$ [7a, 7b].

[4] The rates of Stark mixing were kindly provided to us by V.P. Popov.

3 Results of calculations

In order to verify the cascade scheme of Figure 1 we compare some of its calculated characteristics with the experimental data for pure H_2 and D_2 as well as for their mixtures with He isotopes in a wide range of ϕ and $C_{\rm He}$. The comparison between the calculated and experimental data for relative intensities of K-lines of $p\mu$ and $d\mu$ atoms in pure H_2 and D_2, respectively, and for $q_{1s}^{\rm He}$ is presented. The time-evolution of cascade processes is presented here for the first time, which can be very important for understanding the complex dynamics of muonic cascade. Earlier [2,13], we considered for comparison with experiments (see Ref. [25] in [2]) only a single characteristics of the cascade, $q_{1s}^{\rm He}$, for $H_2 + {}^4$He at $\phi = (2.3 \div 4.5)\%$ and $C_{^4{\rm He}} = 0.05 \div 0.5$. As follows from Figure 6 and Table 2 in [2] the agreement between theoretical and experimental $q_{1s}^{\rm He}$ is possible if one assumes $\varepsilon \sim 2 \div 5$ eV. In the present paper we extend our consideration to a comparison of our calculations of relative intensities of K-lines with the available experimental data. Such comparison requires inclusion of $3 \to 2$ Coulomb deexcitation which was ignored in [2,13].

However, there exists a wide uncertainty in the calculation of Coulomb deexcitation rates obtained by different methods [9a, 9b, 9c]. In our calculations we use different assumptions about the contribution of Coulomb deexcitation to the cascade development by scaling its rate: $\lambda_{\rm C} = \kappa\lambda$, where $\kappa = 1$ corresponds to Coulomb rates obtained in [9a].

We consider a $3 \to 2$ Coulomb deexcitation rate as a free parameter for fitting the calculated relative intensities of K_α, K_β and K_γ to corresponding experimental data in pure H_2 and D_2. The ratio of $4 \to 3$ and $3 \to 2$ rates is also obtained from fits to experimental intensities of the K-lines[5].

The ratios of other Coulomb deexcitation rates were taken from [9c] multiplied by corresponding correction factor. However, they influence only weakly the calculated intensities of K-lines and could be taken, in fact, from any of the paper [9a, 9b, 9c].

The results of our calculations of the different characteristics of muonic hydrogen cascade obtained for hydrogen-helium mixtures with Coulomb deexcitation rates from [9c] are presented in Figures 2–5. Figure 6 illustrate our calculation of K-line intensities for pure H_2 and D_2 with Coulomb deexcitation rates obtained by fitting the calculated results to experimental data.

Figure 2 shows the results of calculations of $q_n^{\rm He}(t, \phi)$ in $D_2 + {}^3$He mixture obtained for $n = 1 \div 12$ and $\varepsilon = 0.04$ eV with $C_{\rm He} = 0.05$.

The number of muons transferred to helium from muonic deuterium, $N_n^{\rm tr}(t, \phi)$, has also been calculated for $\varepsilon = 0.04$ eV and $C_{\rm He} = 0.05$ in $D_2 + {}^3$He (Fig. 3).

Figure 4 shows the results for the relative K X-ray yields *i.e.* $I_{K_\alpha}(t, \phi)$ for $2p \to 1s$ transition, $I_{K_\beta}(t, \phi)$ for $3p \to 1s$ transition, $I_{K_\gamma}(t, \phi)$ for $4p \to 1s$ transition,

[5] Unfortunately the $4 \to 3$ Coulomb deexcitation rate was absent in [9a].

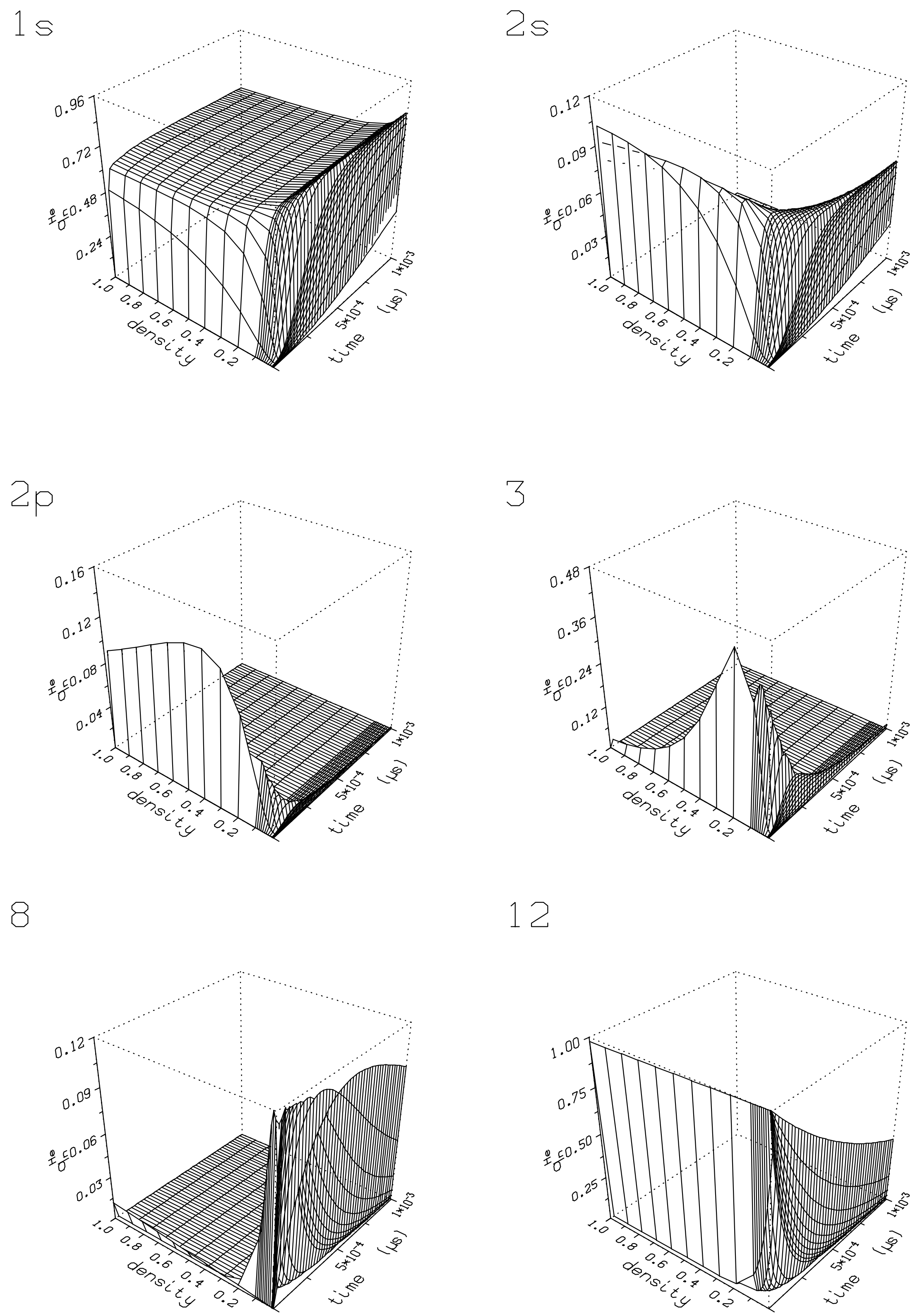

Fig. 2. Dependence of $q_n^{\rm He}(t,\phi)$ for different n, calculated for $\varepsilon = 0.04$ eV and $C_{\rm He} = 0.05$ in $D_2 + {}^3$He mixture; Coulomb deexcitation rates were taken from [9c].

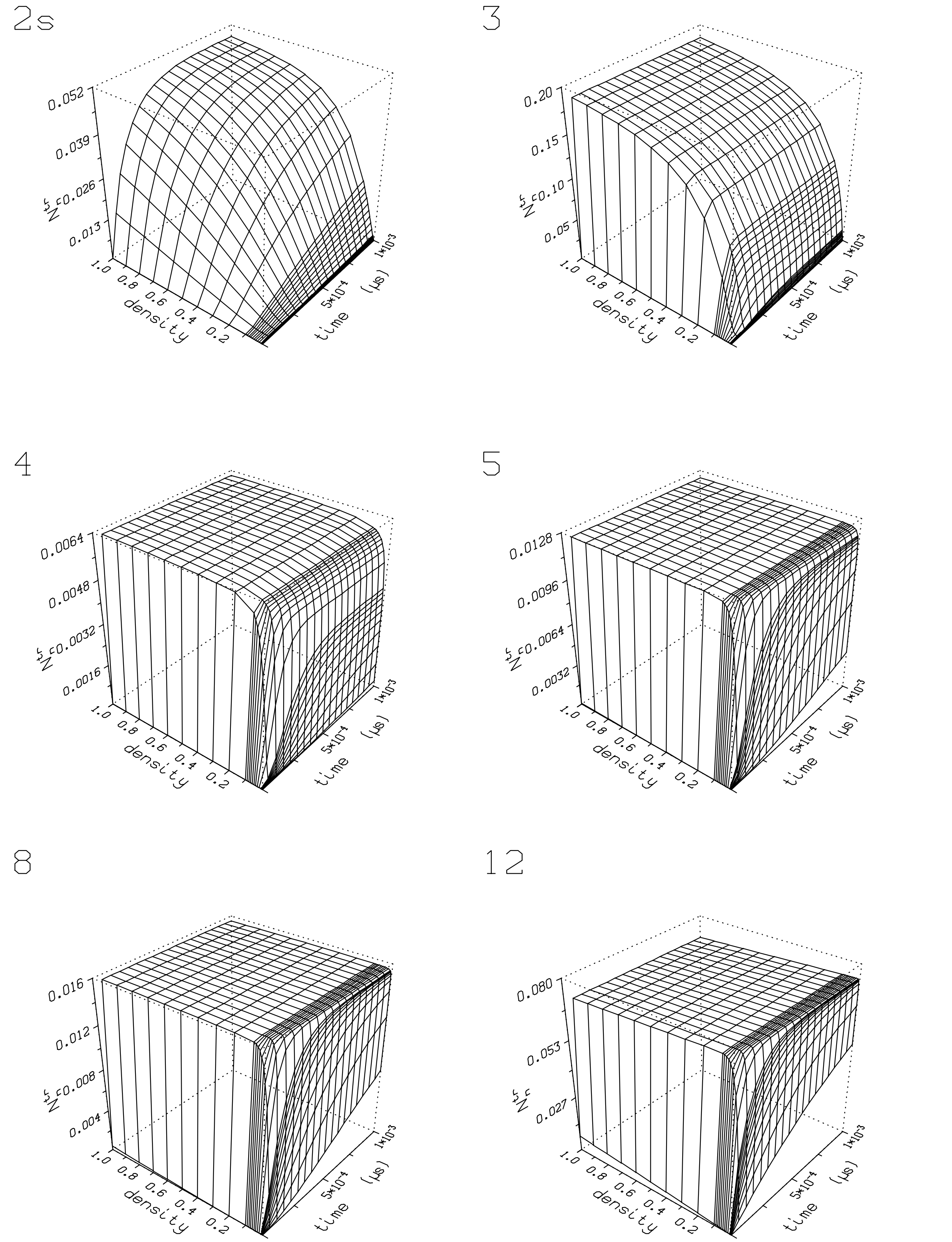

Fig. 3. The number of muons transferred to helium $N_n^{\rm tr}(t,\phi)$ calculated for different n, $\varepsilon = 0.04$ eV and $C_{\rm He} = 0.05$ in $\rm D_2 + {}^3He$. Coulomb deexcitation rates were taken from [9c].

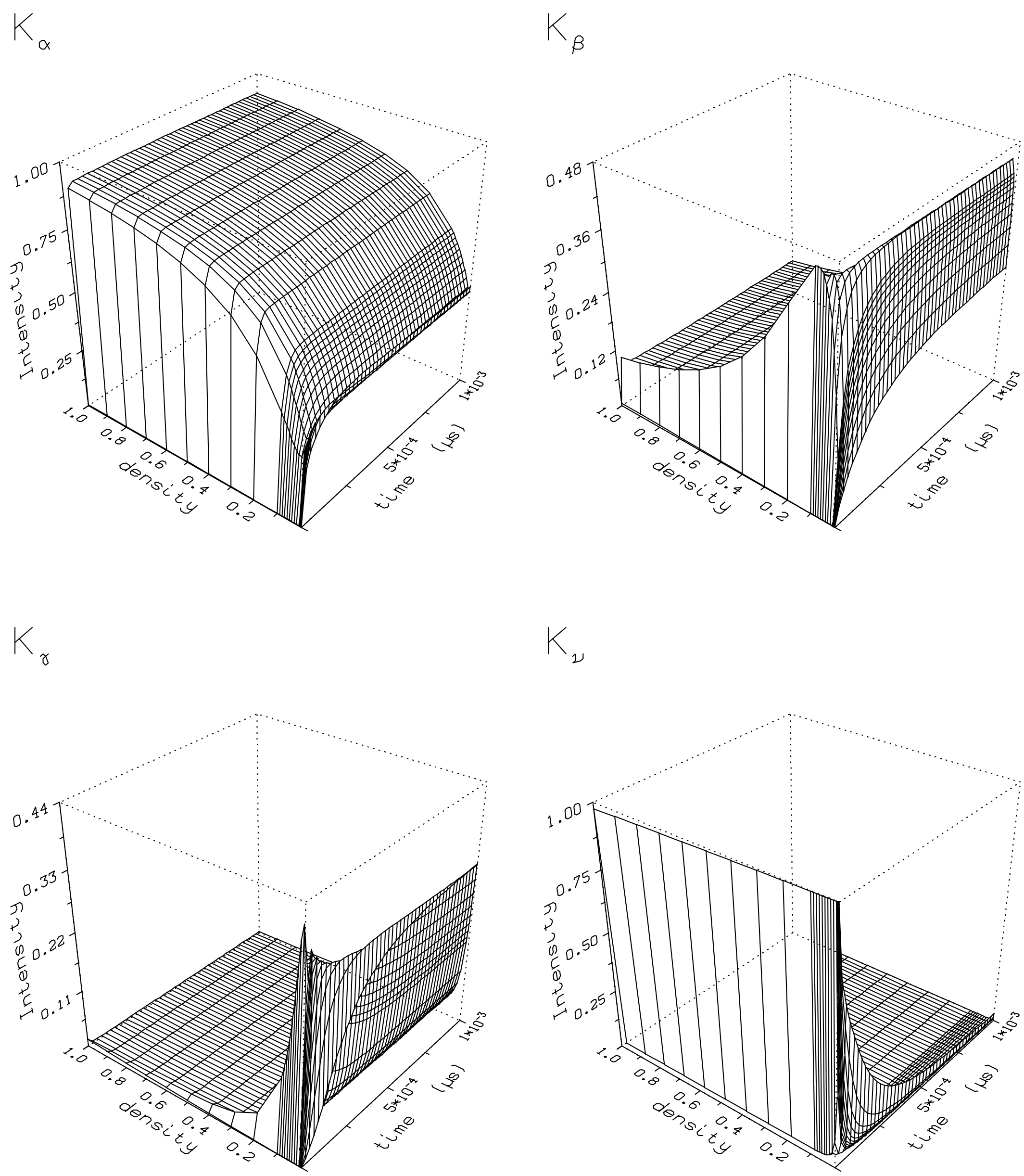

Fig. 4. The ϕ and t dependence of relative intensities of K X-rays calculated for $\varepsilon = 0.04$ eV and $C_{\rm He} = 0.05$ in $D_2 + {}^3$He. Coulomb deexcitation rates were taken from [9c].

and $I_{K_\nu}(t,\phi)$ for $(5p \to 1s) + (6p \to 1s) + (7p \to 1s) + ...$ transitions obtained for $\varepsilon = 0.04$ eV and $C_{\rm He} = 0.05$ in $D_2 + {}^3$He.

Figure 5 demonstrates the agreement of our calculated $q_{1s}^{\rm He}$ with the experimental data for $H_2 + {}^3$He ($\phi = 5.2\%$) and $H_2 + {}^4$He ($\phi = 7.36\%$) mixtures [14].

The experimental $q_{1s}^{\rm He}$ data in $H_2 + {}^4$He and $H_2 + {}^3$He agree with the calculated ones within the experimental errors for $\varepsilon \sim 2$ eV. The comparison for $H_2 + {}^4$He at $\phi = (2.3 \div 4.5)\%$ [2,15] and for $H_2 + D_2$ mixture [16] indicates $\varepsilon \sim 2 \div 5$ eV.

The ϕ-dependence of relative intensities of K-lines in pure H_2 and D_2 in the range $(10^{-5} \div 1)$ is presented in Figure 6a and 6b, respectively, together with the experimental data of references [17–24].

As can be seen in Figure 6a for pure H_2, good agreement between theory and experiment is possible for the rate of Coulomb $3 \to 2$ transition $\sim 9 \times 10^{11}$ s^{-1} (LHD) that exceeds by a factor of about 1.5×10^3 the result of [9c] for $\varepsilon = 1$ eV, by about 100 that of [9b] and by 9 the result of [9a]. If one assumes that muonic hydrogen for $n = 3$ is thermalized *i.e.* $\varepsilon = 0.04$ eV, our $3 \to 2$ Coulomb rate exceeds the result of [9c] by 440 times, that of [9b] by ~ 30 and only by 2.7 that of [9a].

Figure 6b shows us, that for pure D_2, good agreement between theory and experiment is possible for $3 \to 2$ transition rate $\sim 4.8 \times 10^{11}$ s^{-1} that exceeds by $\sim 2 \times 10^4$

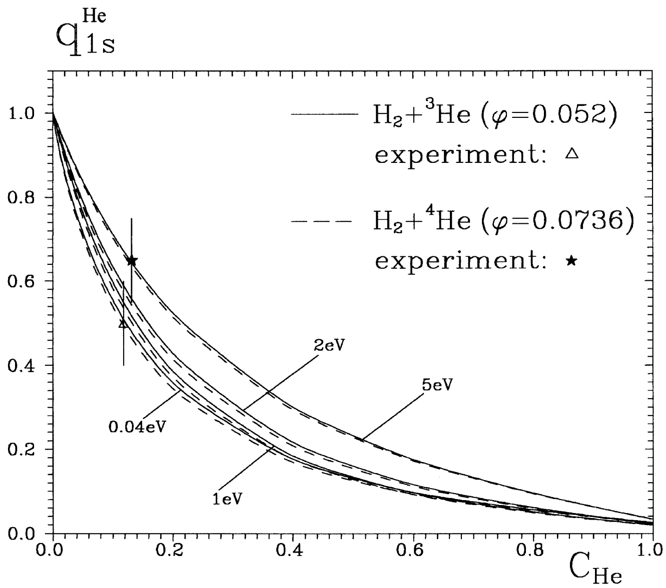

Fig. 5. Comparison of theoretical $q_{1s}^{\rm He}(C_{\rm He})$ curves with experimental data [14] (ε is indicated on curves) obtained for Coulomb rates from [9c].

correspondent result of [9c] for $\varepsilon = 1$ eV and by $\sim 5 \times 10^3$ for $\varepsilon = 0.04$ eV. Unfortunately we cannot compare our results for $d\mu$ with data of [9a] as only muonic hydrogen was considered there.

For $p\mu$ atoms the ratio of Coulomb rates of $3 \to 2$ and $4 \to 3$ transitions is smaller by a factor of 2 compared with $d\mu$ ones. At the same time the Coulomb $3 \to 2$ rate of $p\mu$ atoms is two times larger than that of $d\mu$. The Coulomb transitions for $n > 4$ do not strongly influence the intensities of K-lines. The ratio of Coulomb rates for $3 \to 2$ and $4 \to 3$ transitions contradicts the results of [9c] and is in good agreement with one obtained in [9b].

It is necessary to note that Coulomb deexcitation rates obtained in [9c] (corresponding to $\kappa \sim 0.01$ which gives a contribution to intensities of K-lines coincided, in fact, with $\kappa = 0$) evidently disagree with the experimental data for K_α and K_β intensities. It can be possibly explained by the fact that the method of complex plane is not applicable to nonresonant reaction of Coulomb deexcitation [9c] with great energy gain, which was also discussed in [9b].

The K_γ intensity is not very sensitive to Coulomb deexcitation contribution, which is demonstrated in Figure 3 of [17].

The rates obtained from the analysis of K-line intensities significantly exceed the rates of [9b, 9c] being in better agreement with those of [9a].

The small disagreement of the calculated K_β and K_γ intensities for $p\mu$-atoms with experimental data (see Fig. 6a) for density range $10^{-5} \div 10^{-4}$ and $10^{-2} \div 5 \times 10^{-2}$, respectively, can be explained by the absence in our consideration of any partial Stark transitions between l-sublevels for $n > 3$, which reflects, in fact, the assumption of statistical population of the corresponding l-sublevels.

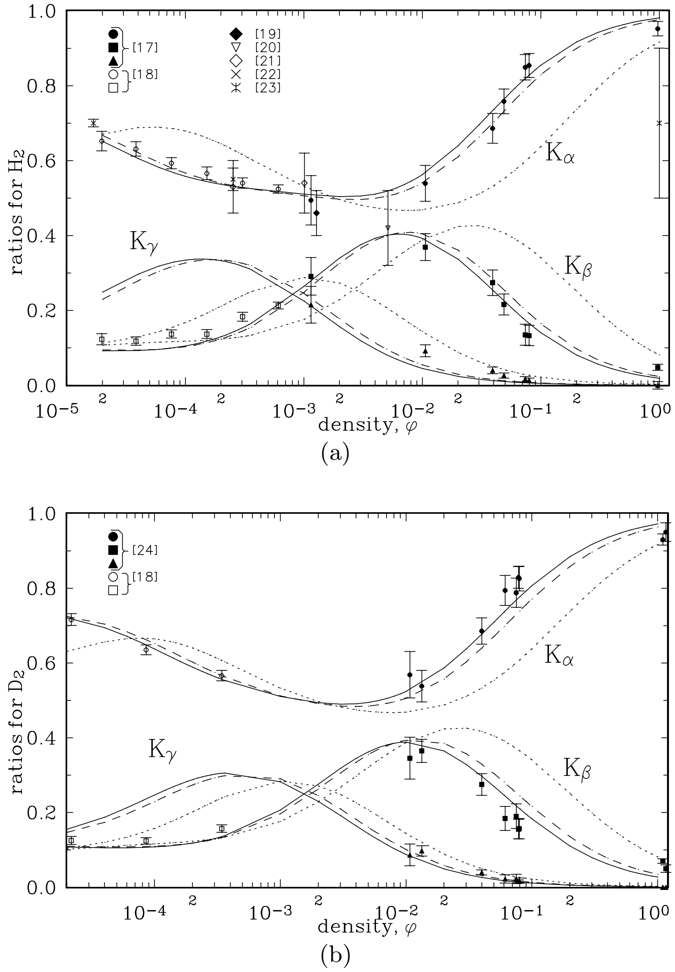

Fig. 6. The ϕ dependence of relative intensities of K X-rays calculated in pure H_2 (a) and pure D_2 (b) for $\phi = 10^{-5} \div 1$ in comparison with experiment [16–24]. The Coulomb deexcitation rates were taken for (a): $\lambda_{\rm C}(3 \to 2) = 1.2 \times 10^{12}$ s^{-1} (solid lines), $\lambda_{\rm C}(3 \to 2) = 9.0 \times 10^{11}$ s^{-1} (long dashed lines), $\lambda_{\rm C}(3 \to 2) = 4.8 \times 10^{9}$ s^{-1} (short dashed lines); (b): $\lambda_{\rm C}(3 \to 2) = 7.2 \times 10^{11}$ s^{-1} (solid lines), $\lambda_{\rm C}(3 \to 2) = 4.8 \times 10^{11}$ s^{-1} (long dashed lines), $\lambda_{\rm C}(3 \to 2) = 4.8 \times 10^{9}$ s^{-1} (short dashed lines).

In our calculations we did not consider partial Coulomb transitions $(n, l) \to (n-1, l-1)$ for $n \geq 4$ using only averaged Coulomb ones with statistical populations of l-sublevels of $n' = n - 1$. Instead of the partial Coulomb transitions $3d \to 2p$, $3p \to 2s$ and $3s \to 2p$ we used averaged $3 \to 2$ rates with a statistical population of $3d$, $3p$ and $3s$.

The Coulomb transitions with $\Delta n = 2, 3...$ should be also taken into account (in addition to $\Delta n = 1$), although their rates are smaller at least by one order of magnitude [9a].

It is necessary to model the initial population of n and l which may significantly influence the populations of l-sublevels for $n = 2 \div 4$, being more important for Coulomb fraction of cascade deexcitation. Information about energy distribution of muonic atoms in the above mentioned states is also important.

The fast $3 \rightarrow 2$ transition can be explained either by resonant mechanism of Coulomb deexcitation [7b] or molecular structure effects.

The $3 \rightarrow 2$ and $4 \rightarrow 3$ Coulomb rates are comparable with the corresponding Auger transitions or exceed them, and are responsible for acceleration of mesic hydrogen [9b] observed in numerous experiments.

In conclusion, we would like to underline that the accurate analysis of main characteristics of muonic cascade in pure H_2 and D_2 as well as in their mixtures with helium isotopes in a wide range of ϕ and $C_{\rm He}$ was performed in the present paper by comparison with the experimental data. For $q_{1s}^{\rm He}$ good agreement between theory and experiment was obtained for $C_{^{3,4}\rm He} = 0 \div 1$ and $\phi = 0 \div 1$ taking into account also the results of the previous paper [2].

Good agreement was also obtained for relative intensities of K_α, K_β and K_γ lines in pure H_2 and D_2. Moreover, the comparison of relative intensities of K_α and K_β lines with experiment enabled us to select Coulomb deexcitation rates in muonic atom cascade mainly in favor of the results of [9a], indicated earlier also by Markushin *et al.* [7a, 7b]. However, our $3 \rightarrow 2$ Coulomb deexcitation rates obtained by fits to experimental data in Figure 6 exceed essentially the theoretical predictions, including [9a]. Possibly, it can be explained by poor information about the cascade parameters which are necessary for very small ϕ used in the fits presented in Figure 6.

At the same time the $q_{1s}^{\rm He}$ is not very sensitive to the values of Coulomb deexcitation rates, especially for small ϕ [7b].

The essential contribution of Coulomb deexcitation process could explain, in natural way, the epithermal contribution in the energy distribution of mesonic hydrogen, observed in numerous experiments (see *e.g.* Ponomarev *et al.* [9b]).

The investigation of dynamics of the muonic cascade in wide range of ϕ and $C_{\rm He}$ using the same experimental method is very important in order to eliminate possible systematic errors. The investigation of dynamics of muonic atom cascade at very small $\phi \sim 10^{-7}$ may enable one to verify the initial stage of the cascade, which is, of course, very important for following stages of cascade. It is also necessary to calculate the rates of Stark mixing and Coulomb deexcitation transitions between l-sublevels for all n. The realization of a multiparameter Monte-Carlo program is important for the analysis of all complex processes determining muonic cascade.

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